Remainder and Factor Theorems

Question 1.

Find, in each case, the remainder when:

(i) $x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.

(ii) $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$.

(iii) $x^4 + 1$ is divided by $x + 1$.

Solution:

By remainder theorem we know that when a polynomial f (x) is divided by x – a, then the remainder is f(a).

(i) $f(x) = x^4 - 3x^2 + 2x + 1$

$\text{Remainder} = f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$

(ii) $f(x) = x^3 + 3x^2 - 12x + 4$

$$\begin{aligned}\text{Remainder} = f(2) &= (2)^3 + 3(2)^2 - 12(2) + 4\\ &= 8 + 12 - 24 + 4\\ &= 0\end{aligned}$$

(iii) $f(x) = x^4 + 1$

$\text{Remainder} = f(-1) = (-1)^4 + 1 = 1 + 1 = 2$

Question 2.

Show that:

(i) $x - 2$ is a factor of $5x^2 + 15x - 50$.

(ii) $3x + 2$ is a factor of $3x^2 - x - 2$.

Solution:

(x – a) is a factor of a polynomial f(x) if the remainder, when f(x) is divided by (x – a), is

0, i.e., if f(a) = 0.

(i) $f(x) = 5x^2 + 15x - 50$

$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$

Hence, $x - 2$ is a factor of $5x^2 + 15x - 50$.

(ii) $f(x) = 3x^2 - x - 2$

$$f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right) - 2 = \frac{4}{3} + \frac{2}{3} - 2 = 2 - 2 = 0$$

Hence, $3x + 2$ is a factor of $3x^2 - x - 2$.

Question 3.
Use the Remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.
(i) $x + 1$
(ii) $2x - 1$
(iii) $x + 2$

Solution:
By remainder theorem we know that when a polynomial f (x) is divided by x – a, then the remainder is f(a).
Let $f(x) = 2x^3 + 3x^2 - 5x - 6$

(i) $f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$
Thus, (x + 1) is a factor of the polynomial f(x).

(ii)

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) - 6$$
$$= \frac{1}{4} + \frac{3}{4} - \frac{5}{2} - 6$$
$$= -\frac{5}{2} - 5 = \frac{-15}{2} \neq 0$$

Thus, (2x – 1) is not a factor of the polynomial f(x).

(iii) $f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$
Thus, (x + 2) is a factor of the polynomial f(x).

Question 4.
(i) If 2x + 1 is a factor of $2x^2 + ax - 3$, find the value of a.
(ii) Find the value of k, if 3x – 4 is a factor of expression $3x^2 + 2x - k$.

Solution:

(i) 2x + 1 is a factor of $f(x) = 2x^2 + ax - 3$.

$$\therefore f\left(\frac{-1}{2}\right) = 0$$

$$\Rightarrow 2\left(\frac{-1}{2}\right)^2 + a\left(\frac{-1}{2}\right) - 3 = 0$$

$$\Rightarrow \frac{1}{2} - \frac{a}{2} = 3$$

$$\Rightarrow 1 - a = 6$$

$$\Rightarrow a = -5$$

(ii) 3x - 4 is a factor of $g(x) = 3x^2 + 2x - k$.

$$\therefore f\left(\frac{4}{3}\right) = 0$$

$$\Rightarrow 3\left(\frac{4}{3}\right)^2 + 2\left(\frac{4}{3}\right) - k = 0$$

$$\Rightarrow \frac{16}{3} + \frac{8}{3} - k = 0$$

$$\Rightarrow \frac{24}{3} = k$$

$$\Rightarrow k = 8$$

Question 5.
Find the values of constants a and b when x – 2 and x + 3 both are the factors of expression $x^3 + ax^2 + bx - 12$.

Solution:

Let $f(x) = x^3 + ax^2 + bx - 12$
$x - 2 = 0 \Rightarrow x = 2$
$x - 2$ is a factor of $f(x)$. So, remainder = 0

$\therefore (2)^3 + a(2)^2 + b(2) - 12 = 0$

$\Rightarrow 8 + 4a + 2b - 12 = 0$

$\Rightarrow 4a + 2b - 4 = 0$

$\Rightarrow 2a + b - 2 = 0 \quad \ldots(1)$

$x + 3 = 0 \Rightarrow x = -3$
$x + 3$ is a factor of $f(x)$. So, remainder = 0

$\therefore (-3)^3 + a(-3)^2 + b(-3) - 12 = 0$

$\Rightarrow -27 + 9a - 3b - 12 = 0$

$\Rightarrow 9a - 3b - 39 = 0$

$\Rightarrow 3a - b - 13 = 0 \quad \ldots(2)$

Adding (1) and (2), we get,
$5a - 15 = 0$
$\Rightarrow a = 3$
Putting the value of a in (1), we get,
$6 + b - 2 = 0$
$\Rightarrow b = -4$

Question 6.
find the value of k, if 2x + 1 is a factor of $(3k + 2)x^3 + (k - 1)$.

Solution:

Let $f(x) = (3k + 2)x^3 + (k - 1)$

$2x + 1 = 0 \Rightarrow x = \frac{-1}{2}$

Since, $2x + 1$ is a factor of $f(x)$, remainder is 0.

$\therefore (3k + 2)\left(\frac{-1}{2}\right)^3 + (k - 1) = 0$

$\Rightarrow \frac{-(3k + 2)}{8} + (k - 1) = 0$

$\Rightarrow \frac{-3k - 2 + 8k - 8}{8} = 0$

$\Rightarrow 5k - 10 = 0$

$\Rightarrow k = 2$

Question 7.
Find the value of a, if x – 2 is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.

Solution:

$f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$
$x - 2 = 0 \Rightarrow x = 2$
Since, x – 2 is a factor of f(x), remainder = 0.
$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$
$64 - 96 - 16a + 24a + 8a + 8 = 0$
$-24 + 16a = 0$
$16a = 24$
$a = 1.5$

Question 8.
Find the values of m and n so that x – 1 and x + 2 both are factors of $x^3 + (3m + 1)x^2 + nx - 18$.

Solution:

Let $f(x) = x^3 + (3m + 1)x^2 + nx - 18$
$x - 1 = 0 \Rightarrow x = 1$
x - 1 is a factor of f(x). So, remainder = 0
$\therefore (1)^3 + (3m + 1)(1)^2 + n(1) - 18 = 0$
$\Rightarrow 1 + 3m + 1 + n - 18 = 0$
$\Rightarrow 3m + n - 16 = 0 \quad \ldots(1)$
$x + 2 = 0 \Rightarrow x = -2$
x + 2 is a factor of f(x). So, remainder = 0
$\therefore (-2)^3 + (3m + 1)(-2)^2 + n(-2) - 18 = 0$
$\Rightarrow -8 + 12m + 4 - 2n - 18 = 0$
$\Rightarrow 12m - 2n - 22 = 0$
$\Rightarrow 6m - n - 11 = 0 \quad \ldots(2)$
Adding (1) and (2), we get,
$9m - 27 = 0$
$m = 3$
Putting the value of m in (1), we get,
$3(3) + n - 16 = 0$
$9 + n - 16 = 0$
$n = 7$

Question 9.
When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k. Find the value of constant k.

Solution:

Let $f(x) = x^3 + 2x^2 - kx + 4$
$x - 2 = 0 \Rightarrow x = 2$
On dividing f(x) by x - 2, it leaves a remainder k.
$\therefore\ f(2) = k$
$(2)^3 + 2(2)^2 - k(2) + 4 = k$
$8 + 8 - 2k + 4 = k$
$20 = 3k$
$k = \frac{20}{3} = 6\frac{2}{3}$

Question 10.
Find the value of a, if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.

Solution:

Let $f(x) = ax^3 + 9x^2 + 4x - 10$
$x + 3 = 0 \Rightarrow x = -3$
On dividing f(x) by x + 3, it leaves a remainder 5.
$\therefore\ f(-3) = 5$
$a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$
$-27a + 81 - 12 - 10 = 5$
$54 = 27a$
$a = 2$

Question 11.
If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of a and b.

Solution:

Let $f(x) = x^3 + ax^2 + bx + 6$

$x - 2 = 0 \Rightarrow x = 2$

Since, x - 2 is a factor, remainder = 0

$\therefore f(2) = 0$

$(2)^3 + a(2)^2 + b(2) + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$2a + b + 7 = 0 \quad \ldots(i)$

$x - 3 = 0 \Rightarrow x = 3$

On dividing f(x) by x - 3, it leaves a remainder 3.

$\therefore f(3) = 3$

$(3)^3 + a(3)^2 + b(3) + 6 = 3$

$27 + 9a + 3b + 6 = 3$

$3a + b + 10 = 0 \quad \ldots(ii)$

Subtracting (i) from (ii), we get,

$a + 3 = 0$

$a = -3$

Substituting the value of a in (i), we get,

$-6 + b + 7 = 0$

$b = -1$

Question 12.

The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b.

Solution:

Let $f(x) = 2x^3 + ax^2 + bx - 2$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

On dividing f(x) by 2x - 3, it leaves a remainder 7.

$\therefore 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7$

$\frac{27}{4} + \frac{9a}{4} + \frac{3b}{2} = 9$

$\frac{27 + 9a + 6b}{4} = 9$

$27 + 9a + 6b = 36$

$9a + 6b - 9 = 0$

$3a + 2b - 3 = 0 \quad \ldots(i)$

$x + 2 = 0 \Rightarrow x = -2$

On dividing f(x) by x + 2, it leaves a remainder 0.

$\therefore 2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$

$-16 + 4a - 2b - 2 = 0$

$4a - 2b - 18 = 0 \quad \ldots(ii)$

Adding (i) and (ii), we get,

$7a - 21 = 0$

$a = 3$

Substituting the value of a in (i), we get,

$3(3) + 2b - 3 = 0$

$9 + 2b - 3 = 0$

$2b = -6$

$b = -3$

Question 13.

What number should be added to $3x^3 - 5x^2 + 6x$ so that when resulting polynomial is divided by $x - 3$, the remainder is 8?

Solution:

Let the number k be added and the resulting polynomial be f(x).

So, $f(x) = 3x^3 - 5x^2 + 6x + k$

It is given that when f(x) is divided by (x - 3), the remainder is 8.

$\therefore f(3) = 8$

$3(3)^3 - 5(3)^2 + 6(3) + k = 8$

$81 - 45 + 18 + k = 8$

$54 + k = 8$

$k = -46$

Thus, the required number is -46.

Question 14.

What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it with $x - 2$, the remainder is 10.

Solution:

Let the number to be subtracted be k and the resulting polynomial be f(x).
So, $f(x) = x^3 + 3x^2 - 8x + 14 - k$
It is given that when f(x) is divided by (x - 2), the remainder is 10.

$\therefore f(2) = 10$

$(2)^3 + 3(2)^2 - 8(2) + 14 - k = 10$

$8 + 12 - 16 + 14 - k = 10$

$18 - k = 10$

$k = 8$

Thus, the required number is 8.

Question 15.

The polynomials $2x^3 - 7x^2 + ax - 6$ and $x^3 - 8x^2 + (2a + 1)x - 16$ leaves the same remainder when divided by x – 2. Find the value of ‘a’.

Solution:

Let $f(x) = 2x^3 - 7x^2 + ax - 6$
$x - 2 = 0 \Rightarrow x = 2$
When f(x) is divided by (x - 2), remainder = f(2)

$\therefore f(2) = 2(2)^3 - 7(2)^2 + a(2) - 6$
$= 16 - 28 + 2a - 6$
$= 2a - 18$

Let $g(x) = x^3 - 8x^2 + (2a + 1)x - 16$
When g(x) is divided by (x - 2), remainder = g(2)

$\therefore g(2) = (2)^3 - 8(2)^2 + (2a + 1)(2) - 16$
$= 8 - 32 + 4a + 2 - 16$
$= 4a - 38$

By the given condition, we have:
f(2) = g(2)
2a - 18 = 4a - 38
4a - 2a = 38 - 18
2a = 20
a = 10
Thus, the value of a is 10.

Question 16.

If (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52, find the values of a and b

Solution:

Since (x − 2) is a factor of polynomial $2x^3 + ax^2 + bx - 14$, we have

$2(2)^3 + a(2)^2 + b(2) - 14 = 0$

$\Rightarrow 16 + 4a + 2b - 14 = 0$

$\Rightarrow 4a + 2b + 2 = 0$

$\Rightarrow 2a + b + 1 = 0$

$\Rightarrow 2a + b = -1 \quad ....(i)$

On dividing by (x − 3), the polynomial $2x^3 + ax^2 + bx - 14$ leaves remainder 52,

$\Rightarrow 2(3)^3 + a(3)^2 + b(3) - 14 = 52$

$\Rightarrow 54 + 9a + 3b - 14 = 52$

$\Rightarrow 9a + 3b + 40 = 52$

$\Rightarrow 9a + 3b = 12$

$\Rightarrow 3a + b = 4 \quad ....(ii)$

Subtracting (i) from (ii), we get

$a = 5$

Substituting a = 5 in (i), we get

$2 \times 5 + b = -1$

$\Rightarrow 10 + b = -1$

$\Rightarrow b = -11$

Hence, a = 5 and b = −11.

Question 17.

Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by x + 3.

Solution:

$x + 3 = 0 \Rightarrow x = -3$

Since, the given polynomials leave the same remainder when divided by (x − 3),

Value of polynomial $ax^3 + 3x^2 - 9$ at x = −3 is same as value of polynomial $2x^3 + 4x + a$ at x = −3.

$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$
$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$
$\Rightarrow -27a + 18 = -66 + a$
$\Rightarrow 28a = 84$
$\Rightarrow a = \frac{84}{28}$
$\Rightarrow a = 3$

Exercise 8B

Question 1.
Using the Factor Theorem, show that:
(i) $(x - 2)$ is a factor of $x^3 - 2x^2 - 9x + 18$. Hence, factorise the expression $x^3 - 2x^2 - 9x + 18$ completely.
(ii) $(x + 5)$ is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.
(iii) $(3x + 2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$. Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.

Solution:
(i) Let $f(x) = x^3 - 2x^2 - 9x + 18$
$x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Remainder = f(2)
$= (2)^3 - 2(2)^2 - 9(2) + 18$
$= 8 - 8 - 18 + 18$
$= 0$
Hence, $(x - 2)$ is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & x^2 - 9 \\ x - 2 & x^3 - 2x^2 - 9x + 18 \\ & x^3 - 2x^2 \\ \hline & -9x + 18 \\ & -9x + 18 \\ \hline & 0 \end{array}$$

$\therefore x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$

(ii) Let $f(x) = 2x^3 + 5x^2 - 28x - 15$
$x + 5 = 0 \Rightarrow x = -5$

$\therefore$ Remainder = f(-5)
$= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15$
$= -250 + 125 + 140 - 15$
$= -265 + 265$
$= 0$
Hence, (x + 5) is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x+5 & 2x^3 + 5x^2 - 28x - 15 \\ & \underline{2x^3 + 10x^2} \\ & -5x^2 - 28x \\ & \underline{-5x^2 - 25x} \\ & -3x - 15 \\ & \underline{-3x - 15} \\ & 0 \end{array}$$

$\therefore 2x^3 + 5x^2 - 28x - 15 = (x + 5)(2x^2 - 5x - 3)$
$= (x + 5)[2x^2 - 6x + x - 3]$
$= (x + 5)[2x(x - 3) + 1(x - 3)]$
$= (x + 5)(2x + 1)(x - 3)$

(iii) Let $f(x) = 3x^3 + 2x^2 - 3x - 2$
$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

$$\therefore \text{Remainder} = f\left(\frac{-2}{3}\right)$$
$$= 3\left(\frac{-2}{3}\right)^3 + 2\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) - 2$$
$$= \frac{-8}{9} + \frac{8}{9} + 2 - 2$$
$$= 0$$
Hence, (3x + 2) is a factor of f(x).

Now, we have:

$$
\begin{array}{r}
x^2 - 1 \\
3x+2 \overline{)\, 3x^3 + 2x^2 - 3x - 2} \\
\underline{3x^3 + 2x^2 \quad\quad\quad} \\
-3x - 2 \\
\underline{-3x - 2} \\
0
\end{array}
$$

$\therefore 3x^3 + 2x^2 - 3x - 2 = (3x+2)(x^2-1) = (3x+2)(x+1)(x-1)$

Question 2.

Using the Remainder Theorem, factorise each of the following completely.

(i) $3x^3 + 2x^2 - 19x + 6$

(ii) $2x^3 + x^2 - 13x + 6$

(iii) $3x^3 + 2x^2 - 23x - 30$

(iv) $4x^3 + 7x^2 - 36x - 63$

(v) $x^3 + x^2 - 4x - 4$

Solution:

(i)

For $x = 2$, the value of the given expression $3x^3 + 2x^2 - 19x + 6$

$= 3(2)^3 + 2(2)^2 - 19(2) + 6$

$= 24 + 8 - 38 + 6$

$= 0$

$\Rightarrow x - 2$ is a factor of $3x^3 + 2x^2 - 19x + 6$

Now let us do long division.

$$
\begin{array}{r}
3x^2 + 8x - 3 \\
x-2 \overline{)\, 3x^3 + 2x^2 - 19x + 6} \\
\underline{3x^3 - 6x^2 \quad\quad\quad\quad} \\
8x^2 - 19x \quad \\
\underline{8x^2 - 16x} \quad \\
-3x + 6 \\
\underline{-3x + 6} \\
0
\end{array}
$$

Thus we have,

$$3x^3 + 2x^2 - 19x + 6 = (x-2)(3x^2 + 8x - 3)$$
$$= (x-2)(3x^2 + 9x - x - 3)$$
$$= (x-2)(3x(x+3) - (x+3))$$
$$= (x-2)(3x-1)(x+3)$$

(ii) Let $f(x) = 2x^3 + x^2 - 13x + 6$
For $x = 2$,
$f(x) = f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$
Hence, $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & 5x^2 - 13x \\ & 5x^2 - 10x \\ \hline & -3x + 6 \\ & -3x + 6 \\ \hline & 0 \end{array}$$

$$\therefore 2x^3 + x^2 - 13x + 6 = (x-2)(2x^2 + 5x - 3)$$
$$= (x-2)(2x^2 + 6x - x - 3)$$
$$= (x-2)[2x(x+3) - (x+3)]$$
$$= (x-2)(x+3)(2x-1)$$

(iii) $f(x) = 3x^3 + 2x^2 - 23x - 30$
For $x = -2$,
$f(x) = f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30$
$= -24 + 8 + 46 - 30 = -54 + 54 = 0$
Hence, $(x + 2)$ is a factor of $f(x)$.

$$
\begin{array}{r}
3x^2 - 4x - 15 \\
x+2\overline{)3x^3 + 2x^2 - 23x - 30} \\
\underline{3x^3 + 6x^2 \quad\quad\quad\quad} \\
-4x^2 - 23x \quad\quad \\
\underline{-4x^2 - 8x \quad\quad} \\
-15x - 30 \\
\underline{-15x - 30} \\
0
\end{array}
$$

$$
\begin{aligned}
\therefore 3x^3 + 2x^2 - 23x - 30 &= (x+2)(3x^2 - 4x - 15) \\
&= (x+2)(3x^2 + 5x - 9x - 15) \\
&= (x+2)[x(3x+5) - 3(3x+5)] \\
&= (x+2)(3x+5)(x-3)
\end{aligned}
$$

(iv) $f(x) = 4x^3 + 7x^2 - 36x - 63$
For $x = 3$,
$f(x) = f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63$
$= 108 + 63 - 108 - 63 = 0$
Hence, $(x + 3)$ is a factor of $f(x)$.

$$
\begin{array}{r}
4x^2 - 5x - 21 \\
x+3\overline{)4x^3 + 7x^2 - 36x - 63} \\
\underline{4x^3 + 12x^2 \quad\quad\quad\quad} \\
-5x^2 - 36x \quad\quad \\
\underline{-5x^2 - 15x \quad\quad} \\
-21x - 63 \\
\underline{-21x - 63} \\
0
\end{array}
$$

$$
\begin{aligned}
\therefore 4x^3 + 7x^2 - 36x - 63 &= (x+3)(4x^2 - 5x - 21) \\
&= (x+3)(4x^2 - 12x + 7x - 21) \\
&= (x+3)[4x(x-3) + 7(x-3)] \\
&= (x+3)(4x+7)(x-3)
\end{aligned}
$$

(v) $f(x) = x^3 + x^2 - 4x - 4$
For $x = -1$,
$f(x) = f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= -1 + 1 + 4 - 4 = 0$
Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & x^2 - 4 \\ x+1 & x^3 + x^2 - 4x - 4 \\ & \underline{x^3 + x^2} \\ & -4x - 4 \\ & \underline{-4x - 4} \\ & 0 \end{array}$$

$\therefore x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$
$= (x + 1)(x + 2)(x - 2)$

Question 3.

Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$.

Solution:

Let $f(x) = 3x^3 + 10x^2 + x - 6$
For $x = -1$,
$f(x) = f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$
Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 3x^2 + 7x - 6 \\ x+1 & 3x^3 + 10x^2 + x - 6 \\ & \underline{3x^3 + 3x^2} \\ & 7x^2 + x \\ & \underline{7x^2 + 7x} \\ & -6x - 6 \\ & \underline{-6x - 6} \\ & 0 \end{array}$$

$$\therefore 3x^3 + 10x^2 + x - 6 = (x+1)(3x^2+7x-6)$$
$$= (x+1)(3x^2+9x-2x-6)$$
$$= (x+1)[3x(x+3)-2(x+3)]$$
$$= (x+1)(x+3)(3x-2)$$

Now, $3x^3 + 10x^2 + x - 6 = 0$

$\Rightarrow (x+1)(x+3)(3x-2) = 0$

$\Rightarrow x = -1, -3, \frac{2}{3}$

Question 4.
Factorise the expression $f(x) = 2x^3 - 7x^2 - 3x + 18$. Hence, find all possible values of x for which $f(x) = 0$.

Solution:

$f(x) = 2x^3 - 7x^2 - 3x + 18$

For $x = 2$,

$f(x) = f(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18$

$= 16 - 28 - 6 + 18 = 0$

Hence, $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 2x^2 - 3x - 9 \\ x - 2 & 2x^3 - 7x^2 - 3x + 18 \\ & 2x^3 - 4x^2 \\ \hline & -3x^2 - 3x \\ & -3x^2 + 6x \\ \hline & -9x + 18 \\ & -9x + 18 \\ \hline & 0 \end{array}$$

$$\therefore 2x^3 - 7x^2 - 3x + 18 = (x-2)(2x^2-3x-9)$$
$$= (x-2)(2x^2-6x+3x-9)$$
$$= (x-2)[2x(x-3)+3(x-3)]$$
$$= (x-2)(x-3)(2x+3)$$

Now, $f(x) = 0$

$\Rightarrow 2x^3 - 7x^2 - 3x + 18 = 0$

$\Rightarrow (x-2)(x-3)(2x+3) = 0$

$\Rightarrow x = 2, 3, \frac{-3}{2}$

Question 5.
Given that x – 2 and x + 1 are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b. Hence, find all the factors of f(x).

Solution:

$f(x) = x^3 + 3x^2 + ax + b$

Since, (x - 2) is a factor of f(x), f(2) = 0

$\Rightarrow (2)^3 + 3(2)^2 + a(2) + b = 0$

$\Rightarrow 8 + 12 + 2a + b = 0$

$\Rightarrow 2a + b + 20 = 0$ ...(i)

Since, (x + 1) is a factor of f(x), f(-1) = 0

$\Rightarrow (-1)^3 + 3(-1)^2 + a(-1) + b = 0$

$\Rightarrow -1 + 3 - a + b = 0$

$\Rightarrow -a + b + 2 = 0$ ...(ii)

Subtracting (ii) from (i), we get,

$3a + 18 = 0$

$\Rightarrow a = -6$

Substituting the value of a in (ii), we get,

$b = a - 2 = -6 - 2 = -8$

$\therefore f(x) = x^3 + 3x^2 - 6x - 8$

Now, for x = -1,

$f(x) = f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$

Hence, (x + 1) is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x + 1 & x^3 + 3x^2 - 6x - 8 \\
 & x^3 + x^2 \\
\hline
 & 2x^2 - 6x \\
 & 2x^2 + 2x \\
\hline
 & -8x - 8 \\
 & -8x - 8 \\
\hline
 & 0
\end{array}
$$

$$
\begin{aligned}
\therefore x^3 + 3x^2 - 6x - 8 &= (x + 1)(x^2 + 2x - 8) \\
&= (x + 1)(x^2 + 4x - 2x - 8) \\
&= (x + 1)[x(x + 4) - 2(x + 4)] \\
&= (x + 1)(x + 4)(x - 2)
\end{aligned}
$$

Question 6.
The expression $4x^3 - bx^2 + x - c$ leaves remainders 0 and 30 when divided by x + 1 and 2x – 3 respectively. Calculate the values of b and c. Hence, factorise the expression completely.

Solution:

Let $f(x) = 4x^3 - bx^2 + x - c$

It is given that when f(x) is divided by (x + 1), the remainder is 0.
$\therefore f(-1) = 0$
$4(-1)^3 - b(-1)^2 + (-1) - c = 0$
$-4 - b - 1 - c = 0$
$b + c + 5 = 0$ ...(i)

It is given that when f(x) is divided by (2x - 3), the remainder is 30.

$$\therefore f\left(\frac{3}{2}\right) = 30$$

$$4\left(\frac{3}{2}\right)^3 - b\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) - c = 30$$

$$\frac{27}{2} - \frac{9b}{4} + \frac{3}{2} - c = 30$$

$$54 - 9b + 6 - 4c - 120 = 0$$

$$9b + 4c + 60 = 0 \qquad \text{...(ii)}$$

Multiplying (i) by 4 and subtracting it from (ii), we get,
$5b + 40 = 0$
$b = -8$

Substituting the value of b in (i), we get,
$c = -5 + 8 = 3$

Therefore, $f(x) = 4x^3 + 8x^2 + x - 3$

Now, for x = -1, we get,
$f(x) = f(-1) = 4(-1)^3 + 8(-1)^2 + (-1) - 3 = -4 + 8 - 1 - 3 = 0$

Hence, (x + 1) is a factor of f(x).

$$
\begin{array}{r|l}
 & 4x^2 + 4x - 3 \\
x+1 & 4x^3 + 8x^2 + x - 3 \\
 & \underline{4x^3 + 4x^2} \\
 & 4x^2 + x \\
 & \underline{4x^2 + 4x} \\
 & -3x - 3 \\
 & \underline{-3x - 3} \\
 & 0
\end{array}
$$

$$
\begin{aligned}
\therefore\ 4x^3 + 8x^2 + x - 3 &= (x+1)(4x^2 + 4x - 3) \\
&= (x+1)(4x^2 + 6x - 2x - 3) \\
&= (x+1)[2x(2x+3) - (2x+3)] \\
&= (x+1)(2x+3)(2x-1)
\end{aligned}
$$

Question 7.

If x + a is a common factor of expressions f(x) = x^2 + px + q and g(x) = x^2 + mx + n;

show that: $a = \frac{n-q}{m-p}$

Solution:

$f(x) = x^2 + px + q$

It is given that (x + a) is a factor of f(x).

$\therefore\ f(-a) = 0$

$\Rightarrow (-a)^2 + p(-a) + q = 0$

$\Rightarrow a^2 - pa + q = 0$

$\Rightarrow a^2 = pa - q \qquad \ldots(i)$

$g(x) = x^2 + mx + n$

It is given that (x + a) is a factor of g(x).

$\therefore\ g(-a) = 0$

$\Rightarrow (-a)^2 + m(-a) + n = 0$

$\Rightarrow a^2 - ma + n = 0$

$\Rightarrow a^2 = ma - n \qquad \ldots(ii)$

From (i) and (ii), we get,

$pa - q = ma - n$
$n - q = a(m - p)$
$$a = \frac{n-q}{m-p}$$
Hence, proved.

Question 8.
The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$, when divided by $x - 4$, leave the same remainder in each case. Find the value of a.

Solution:
Let $f(x) = ax^3 + 3x^2 - 3$
When f(x) is divided by (x – 4), remainder = f(4)
$f(4) = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

Let $g(x) = 2x^3 - 5x + a$
When g(x) is divided by (x – 4), remainder = g(4)
$g(4) = 2(4)^3 - 5(4) + a = a + 108$

It is given that f(4) = g(4)
64a + 45 = a + 108
63a = 63
a = 1

Question 9.
Find the value of 'a', if (x – a) is a factor of $x^3 - ax^2 + x + 2$.

Solution:
Let $f(x) = x^3 - ax^2 + x + 2$
It is given that (x – a) is a factor of f(x).
Remainder = f(a) = 0
$a^3 - a^3 + a + 2 = 0$
a + 2 = 0
a = -2

Question 10.
Find the number that must be subtracted from the polynomial $3y^3 + y^2 - 22y + 15$, so that the resulting polynomial is completely divisible by y + 3.

Solution:
Let the number to be subtracted from the given polynomial be k.
Let $f(y) = 3y^3 + y^2 - 22y + 15 - k$
It is given that f(y) is divisible by (y + 3).

Remainder = f(-3) = 0
$3(-3)^3 + (-3)^2 - 22(-3) + 15 - k = 0$

$-81 + 9 + 66 + 15 - k = 0$
$9 - k = 0$
$k = 9$

Exercise 8C

Question 1.
Show that (x – 1) is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the given expression.

Solution:

Let $f(x) = x^3 - 7x^2 + 14x - 8$
$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$
Hence, (x - 1) is a factor of f(x).

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ x - 1 & x^3 - 7x^2 + 14x - 8 \\ & \underline{x^3 - x^2} \\ & -6x^2 + 14x \\ & \underline{-6x^2 + 6x} \\ & 8x - 8 \\ & \underline{8x - 8} \\ & 0 \end{array}$$

$$\therefore x^3 - 7x^2 + 14x - 8 = (x-1)(x^2 - 6x + 8)$$
$$= (x-1)(x^2 - 2x - 4x + 8)$$
$$= (x-1)[x(x-2) - 4(x-2)]$$
$$= (x-1)(x-2)(x-4)$$

Question 2.

Using Remainder Theorem, factorise:
$x^3 + 10x^2 - 37x + 26$ completely.

Solution:

By Remainder Theorem,

For $x = 1$, the value of the given expression is the remainder.

$x^3 + 10x^2 - 37x + 26$

$= (1)^3 + 10(1)^2 - 37(1) + 26$

$= 1 + 10 - 37 + 26$

$= 37 - 37$

$= 0$

$\Rightarrow x - 1$ is a factor of $x^3 + 10x^2 - 37x + 26$.

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & x^3 - x^2 \\
\hline
 & 11x^2 - 37x \\
 & 11x^2 - 11x \\
\hline
 & -26x + 26 \\
 & -26x + 26 \\
\hline
 & 0
\end{array}
$$

$\therefore\ x^3 + 10x^2 - 37x + 26 = (x - 1)(x^2 + 11x - 26)$

$= (x - 1)(x^2 + 13x - 2x - 26)$

$= (x - 1)[x(x + 13) - 2(x + 13)]$

$\therefore\ x^3 + 10x^2 - 37x + 26 = (x - 1)(x + 13)(x - 2)$

Question 3.

When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m.

Solution:

Let $f(x) = x^3 + 3x^2 - mx + 4$

According to the given information,

$f(2) = m + 3$

$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$

$8 + 12 - 2m + 4 = m + 3$

$24 - 3 = m + 2m$

$3m = 21$

$m = 7$

Question 4.

What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?

Solution:

Let the required number be k.

Let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$

According to the given information,

$f(-2) = 0$

$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$

$-24 - 32 - 8 - 3 - k = 0$

$-67 - k = 0$

$k = -67$

Thus, the required number is -67.

Question 5.

If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b. And then, factorise the given expression completely.

Solution:

Let $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$

Since, $(x + 1)$ is a factor of $f(x)$.

Remainder = $f(-1) = 0$

$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$

$-1 + (a + 1) + (b - 2) - 6 = 0$

$a + b - 8 = 0$ …(i)

Since, $(x - 2)$ is a factor of $f(x)$.

Remainder = $f(2) = 0$

$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$

$8 + 4a + 4 - 2b + 4 - 6 = 0$

$4a - 2b + 10 = 0$

$2a - b + 5 = 0$ …(ii)

Adding (i) and (ii), we get,

$3a - 3 = 0$

$a = 1$

Substituting the value of a in (i), we get,

$1 + b - 8 = 0$

$b = 7$

$f(x) = x^3 + 2x^2 - 5x - 6$

Now, $(x + 1)$ and $(x - 2)$ are factors of $f(x)$. Hence, $(x + 1)(x - 2) = x^2 - x - 2$ is a factor of f(x).

$$
\begin{array}{r|l}
 & x + 3 \\
x^2 - x - 2 & x^3 + 2x^2 - 5x - 6 \\
 & \underline{x^3 - x^2 - 2x} \\
 & 3x^2 - 3x - 6 \\
 & \underline{3x^2 - 3x - 6} \\
 & 0
\end{array}
$$

$f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

Question 6.
If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b.

Solution:
Let $f(x) = x^2 + ax + b$
Since, $(x - 2)$ is a factor of f(x).
Remainder = f(2) = 0
$(2)^2 + a(2) + b = 0$
$4 + 2a + b = 0$
$2a + b = -4$ …(i)
It is given that:
$a + b = 1$ …(ii)
Subtracting (ii) from (i), we get,
$a = -5$
Substituting the value of a in (ii), we get,
$b = 1 - (-5) = 6$

Question 7.
Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.

Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$
For $x = -1$
$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$
$= -1 + 6 - 11 + 6 = 12 - 12 = 0$
Hence, $(x + 1)$ is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
x+1 & x^3 + 6x^2 + 11x + 6 \\
 & x^3 + x^2 \\
\hline
 & 5x^2 + 11x \\
 & 5x^2 + 5x \\
\hline
 & 6x + 6 \\
 & 6x + 6 \\
\hline
 & 0
\end{array}
$$

$$
\begin{aligned}
\therefore x^3 + 6x^2 + 11x + 6 &= (x+1)(x^2 + 5x + 6) \\
&= (x+1)(x^2 + 2x + 3x + 6) \\
&= (x+1)[x(x+2) + 3(x+2)] \\
&= (x+1)(x+2)(x+3)
\end{aligned}
$$

Question 8.

Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.

Solution:

Let $f(x) = mx^3 + 2x^2 - 3$

$g(x) = x^2 - mx + 4$

It is given that f(x) and g(x) leave the same remainder when divided by (x – 2).

Therefore, we have:

$f(2) = g(2)$

$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$

$8m + 8 - 3 = 4 - 2m + 4$

$10m = 3$

$m = 3/10$

Question 9.

The polynomial $px^3 + 4x^2 - 3x + q$ is completely divisible by $x^2 - 1$; find the values of p and q. Also, for these values of p and q factorize the given polynomial completely.

Solution:

Let $f(x) = px^3 + 4x^2 - 3x + q$

It is given that f(x) is completely divisible by $(x^2 - 1) = (x + 1)(x - 1)$.

Therefore, $f(1) = 0$ and $f(-1) = 0$

$f(1) = p(1)^3 + 4(1)^2 - 3(1) + q = 0$

$p + q + 1 = 0$ ...(i)
$f(-1) = p(-1)^3 + 4(-1)^2 - 3(-1) + q = 0$
$-p + q + 7 = 0$ ...(ii)

Adding (i) and (ii), we get,
$2q + 8 = 0$
$q = -4$

Substituting the value of q in (i), we get,
$p = -q - 1 = 4 - 1 = 3$
$f(x) = 3x^3 + 4x^2 - 3x - 4$

Given that f(x) is completely divisible by $(x^2 - 1)$.

$$\begin{array}{r|l} & 3x + 4 \\ x^2 - 1 & 3x^3 + 4x^2 - 3x - 4 \\ & 3x^3 \qquad\quad - 3x \\ \hline & \quad 4x^2 \qquad - 4 \\ & \quad 4x^2 \qquad - 4 \\ \hline & \qquad 0 \end{array}$$

$$\therefore 3x^3 + 4x^2 - 3x - 4 = (x^2 - 1)(3x + 4)$$
$$= (x - 1)(x + 1)(3x + 4)$$

Question 10.
Find the number which should be added to $x^2 + x + 3$ so that the resulting polynomial is completely divisible by $(x + 3)$.

Solution:
Let the required number be k.
Let $f(x) = x^2 + x + 3 + k$
It is given that f(x) is divisible by $(x + 3)$.

Remainder = 0
$f(-3) = 0$
$(-3)^2 + (-3) + 3 + k = 0$
$9 - 3 + 3 + k = 0$
$9 + k = 0$
$k = -9$
Thus, the required number is -9.

Question 11.
When the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A and when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B. Find the value of 'a' if $2A + B = 0$.

Solution:
It is given that when the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A.
$(1)^3 + 2(1)^2 - 5a(1) - 7 = A$
$1 + 2 - 5a - 7 = A$
$-5a - 4 = A$ ...(i)

It is also given that when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B.
$x^3 + ax^2 - 12x + 16 = B$
$(-2)^3 + a(-2)^2 - 12(-2) + 16 = B$
$-8 + 4a + 24 + 16 = B$
$4a + 32 = B$ ...(ii)

It is also given that $2A + B = 0$
Using (i) and (ii), we get,
$2(-5a - 4) + 4a + 32 = 0$
$-10a - 8 + 4a + 32 = 0$
$-6a + 24 = 0$
$6a = 24$
$a = 4$

Question 12.
$(3x + 5)$ is a factor of the polynomial $(a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$. Find the value of 'a', factorise the given polynomial completely.

Solution:

Let $f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$
It is given that $(3x + 5)$ is a factor of $f(x)$.
$\therefore$ Remainder = 0

$$f\left(\frac{-5}{3}\right) = 0$$

$$(a-1)\left(\frac{-5}{3}\right)^3 + (a+1)\left(\frac{-5}{3}\right)^2 - (2a+1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$(a-1)\left(\frac{-125}{27}\right) + (a+1)\left(\frac{25}{9}\right) - (2a+1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$\frac{-125(a-1)+75(a+1)+45(2a+1)-405}{27}=0$$

$$-125a+125+75a+75+90a+45-405=0$$

$$40a-160=0$$

$$40a=160$$

$$a=4$$

$\therefore f(x) = (a-1)x^3 + (a+1)x^2 - (2a+1)x - 15$

$= 3x^3 + 5x^2 - 9x - 15$

$$\begin{array}{r|l} & x^2 - 3 \\ 3x+5 & 3x^3 + 5x^2 - 9x - 15 \\ & 3x^3 + 5x^2 \\ \hline & -9x - 15 \\ & -9x - 15 \\ \hline & 0 \end{array}$$

$$\therefore 3x^3 + 5x^2 - 9x - 15 = (3x+5)(x^2-3)$$

$$= (3x+5)(x+\sqrt{3})(x-\sqrt{3})$$

Question 13.
When divided by x – 3 the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.

Solution:
If (x – 3) divides $f(x) = x^3 - px^2 + x + 6$, then,

Remainder = $f(3) = 3^3 - p(3)^2 + 3 + 6 = 36 - 9p$
If (x – 3) divides $g(x) = 2x^3 - x^2 - (p + 3)x - 6$, then

Remainder = $g(3) = 2(3)^3 - (3)^2 - (p + 3)(3) - 6 = 30 - 3p$
Now, f(3) = g(3)

$\Rightarrow 36 - 9p = 30 - 3p$
$\Rightarrow -6p = -6$
$\Rightarrow p = 1$

Question 14.
Use the Remainder Theorem to factorise the following expression:
$2x^3 + x^2 - 13x + 6$

Solution:

$f(x) = 2x^3 + x^2 - 13x + 6$

Factors of constant term 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Putting x = 2, we have:

$f(2) = 2(2)^3 + 2^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$

Hence (x - 2) is a factor of f(x).

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ x - 2 & 2x^3 + x^2 - 13x + 6 \\ & \underline{2x^3 - 4x^2} \\ & \quad 5x^2 - 13x \\ & \quad \underline{5x^2 - 10x} \\ & \qquad -3x + 6 \\ & \qquad \underline{-3x + 6} \\ & \qquad\quad 0 \end{array}$$

$$\begin{aligned} 2x^3 + x^2 - 13x + 6 &= (x-2)(2x^2 + 5x - 3) \\ &= (x-2)(2x^2 + 6x - x - 3) \\ &= (x-2)(2x(x+3) - 1(x+3)) \\ &= (x-2)(2x-1)(x+3) \end{aligned}$$

Question 15.

Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7.

Solution:

Let $f(x) = 2x^3 + 3x^2 - kx + 5$

Using Remainder Theorem, we have

$f(2) = 7$

$\therefore 2(2)^3 + 3(2)^2 - k(2) + 5 = 7$

$\therefore 16 + 12 - 2k + 5 = 7$

$\therefore 33 - 2k = 7$

$\therefore 2k = 26$

$\therefore k = 13$

Question 16.

What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor?

Solution:
Here, $f(x) = 16x^3 - 8x^2 + 4x + 7$
Let the number subtracted be k from the given polynomial f(x).

Given that 2x + 1 is a factor of f(x).

$$\therefore f\left(-\frac{1}{2}\right) = 0$$
$$\Rightarrow 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$$
$$\Rightarrow 16 \times \left(-\frac{1}{8}\right) - 8 \times \frac{1}{4} - 2 + 7 - k = 0$$
$$\Rightarrow -2 - 2 - 2 + 7 - k = 0$$
$$\Rightarrow -6 + 7 - k = 0$$
$$\Rightarrow k = 1$$

Therefore 1 must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has 2x + 1 as a factor.